2020.04.29(Wed)

EC5102 Midterm Examination

1. For matrices $A = \begin{bmatrix} 2 & 4 \\ 4 & 11 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 4 & 0 \\ 4 & 12 & 4 \\ 0 & 4 & 0 \end{bmatrix}$,
   (a) Factorize each matrix to L and U: Hint: Gaussian elimination process.
   (b) What are the corresponding elementary matrices such as E^{-1}, F^{-1}, and G^{-1}, composing L? Hint: You may use Gauss-Jordan method to find out inverse matrix.
   (c) Prove your answers in (b). Hint: show, e.g., A=LU.
2. (a) Write down the 3x3 finite-difference matrix for $-\frac{d^2u}{dx^2} + u = x$. u(0)=u(1)=0 and h=1/4.
   (b) Denote the h segmentation in the range of x.
3. The reflection matrix along a straight line at an angle is given by:
   $H_{angle} = \begin{bmatrix} 2c^2 - 1 & 2cs \\ 2cs & 2s^2 - 1 \end{bmatrix}$, where c and s are cos and sin function, respectively.
   Prove the product of two different reflection matrices results in a rotation matrix. Hint: you may prove it with $H_\theta H_\alpha$.
4. Solve Ax=b by least squares. Find out best $\hat{x}$, projection point p, and error vector e perpendicular to p.
   $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.
5. From the nonorthogonal vectors a, b, and c,
   $a = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$, $b = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $c = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$.
   (a) Find orthogonal vectors q_1, q_2, and q_3.
   (b) Factorize A to QR, where
   $A = [a \;\; b \;\; c]$ and $Q = [q_1 \;\; q_2 \;\; q_3]$. Hint: find out R.