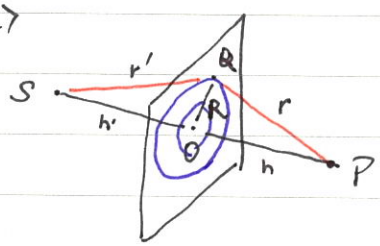
5.5. Fresnel Diffraction

· $\frac{1}{2}\left(\frac{1}{d'}+\frac{1}{d}\right)\delta^2 \ll \lambda$

· Not a plane wave anymore

< Fresnel Zone >



· $r + r' = \sqrt{h^2+R^2} + \sqrt{h'^2+R^2}$

$= h\left[1+\left(\frac{R}{h}\right)^2\right]^{\frac{1}{2}} + h'\left[1+\left(\frac{R}{h'}\right)\right]^{\frac{1}{2}}$

$\doteq h\left(1+\frac{1}{2}\left(\frac{R}{h}\right)^2+\cdots\right) + h'\left(1+\left(\frac{R}{h'}\right)^2+\cdots\right)$

$= h + h' + \frac{1}{2}R^2\left(\frac{1}{h}+\frac{1}{h'}\right)+\cdots$

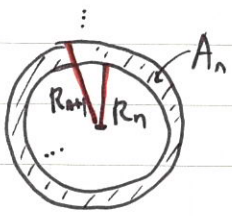
· Divide the region of the aperature such at $(r+r')$ differs by $\frac{\lambda}{2}$. → concentric rings.

→ Fresnel zones

· $\frac{1}{2}R_n^2\left(\frac{1}{h}+\frac{1}{h'}\right) = \frac{\lambda}{2}\cdot n$, $n = 1, 2, 3, \cdots$

Let $\frac{1}{h}+\frac{1}{h'} = \frac{1}{L}$

then $R_n = \sqrt{n\lambda L}$



· $A_n = \pi\left(R_{n+1}^2 - R_n^2\right)$

$= \pi\lambda L[(n+1)-n] = \pi\lambda L$.

→ $A_n = A_{n+1}$: same area!

· Diffraction pattern at P : superposition of A_n.

morning glory

If $A_n = u_n$, (in a small R)

$|U_p| = |u_1| + |u_2| + |u_3| + \cdots$

ex) A circular aperature centered at O & the aperture includes n Fresnel zones.
Because zones have an equal area,
$|U|$ is also equal for all zones.

Q. What is U_p if n is even?

A. $u_n = -u_{n-1}$ due to the π-phase shift.
$\therefore U_p = 0$ by all cancellation.

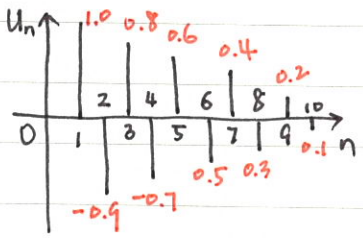
Q. What is U_p if n is odd?

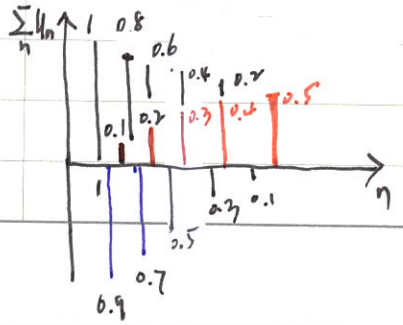
A.

$\left(\frac{1}{rr'}\right)$

• Actual situation : radial distance factor in (5.11)
→ $u_n < u_{n-1}$, If $n \to \infty$, $u_n \to 0$
$n \to \infty$ means that no aperture ($R_n \to \infty$)

Q what is $U_p = ?$





$U_p = u_1 + u_2 + u_3 + u_4 + u_5 + u_6 + u_7 + u_8 + u_9 + u_{10}$

$= 1 - 0.9 + 0.8 - 0.7 + 0.6 - 0.5 + 0.4 - 0.3 + 0.2 - 0.1 + 0$

$= (1 + 0.8 + 0.6 + 0.4 + 0.2) - (0.9 + 0.7 + 0.5 + 0.3 + 0.1)$

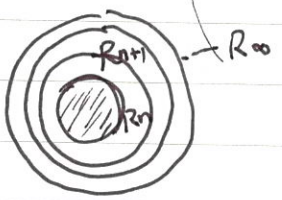
$= 3.0 - 2.5 = 0.5$

$= U_0/2$

morning glory

· Now suppose an obstacle instead of an aperture.

Q. What is $|U_p|^2$?

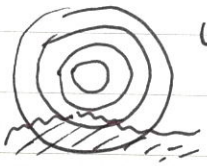


· As discussed above,

$U_p = U_{n+1} + U_{n+2} + U_{n+3} + \cdots$

$= \frac{1}{2} U_{n+1}$

→ The center of diffraction pattern is (bright / dark).



(partial edge block)

Q. What if the aperture is blocked by shadow, what is the effect on U_p?

→ The higher terms in U_n are affected

→ Nearly no effect on U_p.

(major block)

Q. What if the shadow covers the center region?

→ Diminish at both ends of U_n

→ Almost complete cancellation if U_p is along the center.

* if U_p is along the bright (unobstacle) region?

→ Still bright

<Zone plate>

· The obstacle blocks alternatively the zone areas.

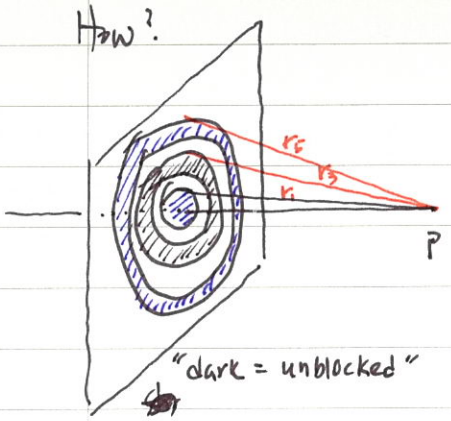
→ (i) even numbered zone blocking;

$|U_p| = |U_1| + |U_3| + |U_5| + \cdots$

→ much brighter than no blocking case. ($\frac{1}{2}|U_1|$)

morning glory

How?



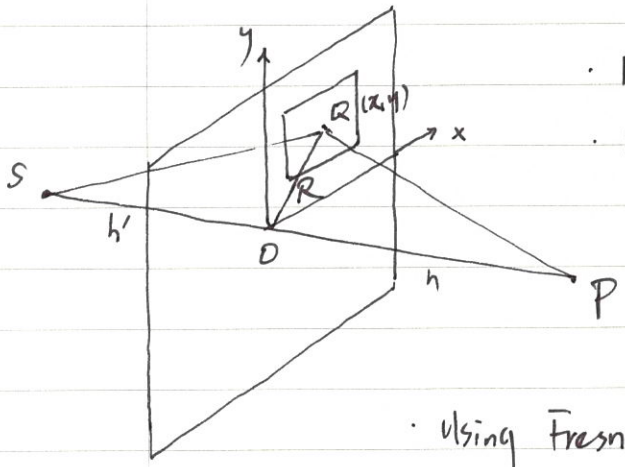
· Due to no effect by $|U_{2n}|$ the zone plate acts as a focal lens.

$$\therefore \frac{1}{L} = \left(\frac{1}{h} + \frac{1}{h'}\right)$$

$$\rightarrow R_n = \sqrt{n\lambda L}$$

$$L = \frac{R_1}{\lambda} \quad (\equiv f)$$

< Rectangular Aperture >



· $R^2 = x^2 + y^2$ from (5.35)

· $r' + r = h + h' + \frac{1}{2}R^2\left(\frac{1}{h} + \frac{1}{h'}\right)$

$$= h + h' + \frac{1}{2L}(x^2 + y^2)$$

· Using Fresnel-Kirchhoff formula (5.11),

$$U_P = C\int_{x_1}^{x_2}\int_{y_1}^{y_2} e^{ik(x^2+y^2)/2L}\,dx\,dy \qquad : e^{ik(r+r')}$$

$$= C\int_{x_1}^{x_2} e^{ikx^2/2L}\,dx \int_{y_1}^{y_2} e^{iky^2/2L}\,dy$$

Using dimensionless variables u and v :

$$u = x\sqrt{\frac{k}{\pi L}} \; ; \; v = y\sqrt{\frac{k}{\pi L}}$$

$$= x\sqrt{\frac{2}{\lambda L}} \qquad = y\sqrt{\frac{2}{\lambda L}}$$

morning glory

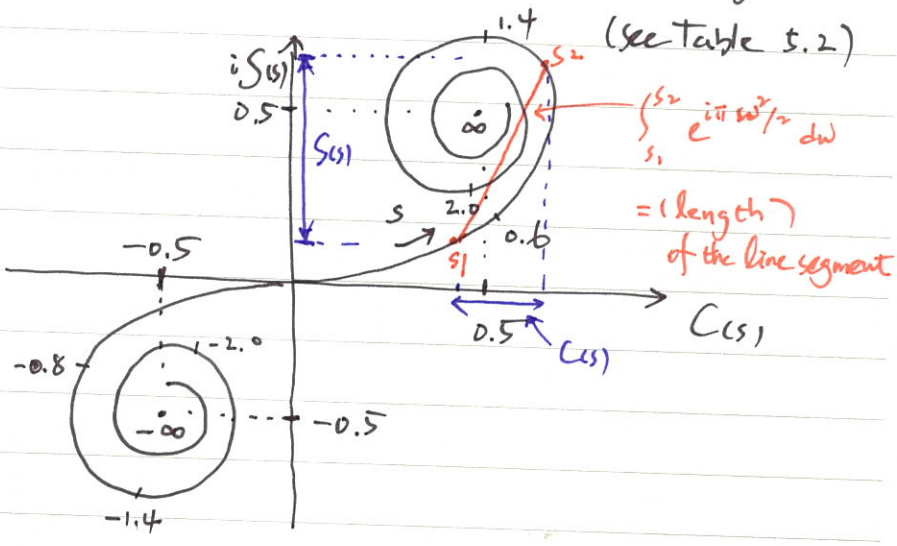
$$U_p = U_1 \int_{u_1}^{u_2} e^{i\pi u^2/2}\,du \int_{v_1}^{v_2} e^{i\pi v^2/2}\,dv, \qquad (5.44)$$

$$U_1 = C\pi L/k.$$

$$\int_0^s e^{i\pi w^2/2}\,dw = C(s) + iS(s),$$

where $C(s) = \int_0^s \cos(\pi w^2/2)\,dw$

$S(s) = \int_0^s \sin(\pi w^2/2)\,dw$ } Fresnel integrals (see Table 5.2)



< Cornu spiral >

Extreme case: $s \to \infty$ (no aperture)

$$C(\infty) = \frac{1}{2} = S(\infty)$$

$$S(-\infty) = -\frac{1}{2} = +C(-\infty)$$

On the cornu spiral, it is the length between $-\infty$ and ∞.

From (5.44)

$$U_p^{\infty} = U_1(1+i)^2 \equiv U_0$$

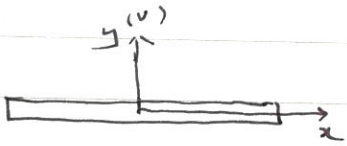
Then, with an aperture,

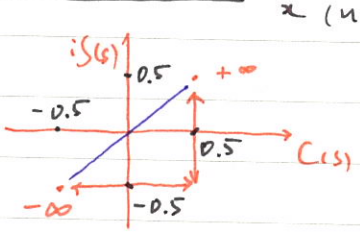
$$U_p = \frac{U_0}{(1+i)^2}\,[C(u)+iS(u)]\Big|_{u_1}^{u_2}\,[C(v)+iS(v)]\Big|_{v_1}^{v_2} \qquad (5.47)$$

morning glory

< Slit and Straight edge >

- Long slit : treated as $u_1 = -\infty$, $u_2 = \infty$



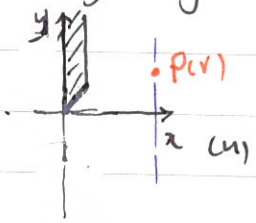


$$\cdot\ U_p = \frac{U_0}{(1+i)^2}\left[C(u)+iS(u)\right]\Big|_{u_1}^{u_2}\left[C(v)+iS(v)\right]\Big|_{v_1}^{v_2}$$

$$= \frac{U_0}{(1+i)^2}(1+i)\left[C(v)+iS(v)\right]\Big|_{v_1}^{v_2}$$

$$= \frac{U_0}{(1+i)}\left[C(v)+iS(v)\right]_{v_1}^{v_2}$$

- Straight edge

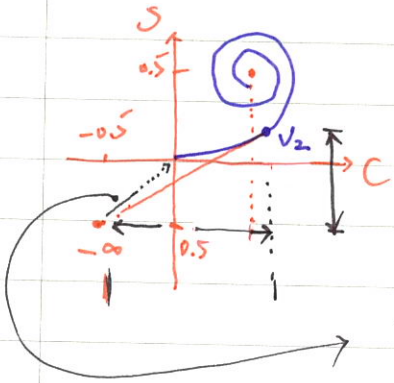


$\cdot\ v_1 = -\infty$; $u_1 = +\infty$, $u_2 = \infty$

$$\cdot\ U_p = \frac{U_0}{(1+i)^2}\underbrace{\left[C(u)+iS(u)\right]\Big|_{u_1}^{u_2}}_{(1+i)}\left[C(v)+iS(v)\right]\Big|_{-\infty}^{v_2}$$

$$= \frac{U_0}{(1+i)}\left[C(v)+iS(v)\right]\Big|_{-\infty}^{v_2}$$

$$= \frac{U_0}{(1+i)}\left[C(v_2)+iS(v_2)+0.5+0.5i\right]$$



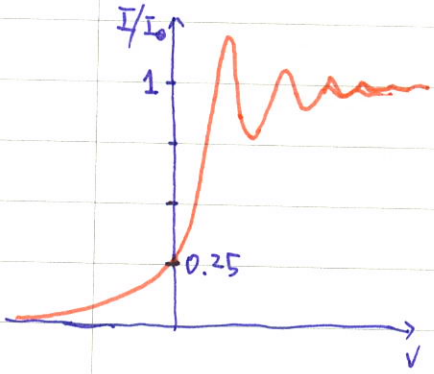
- If P is in the x-axis,

$P(v_2 = 0)$,

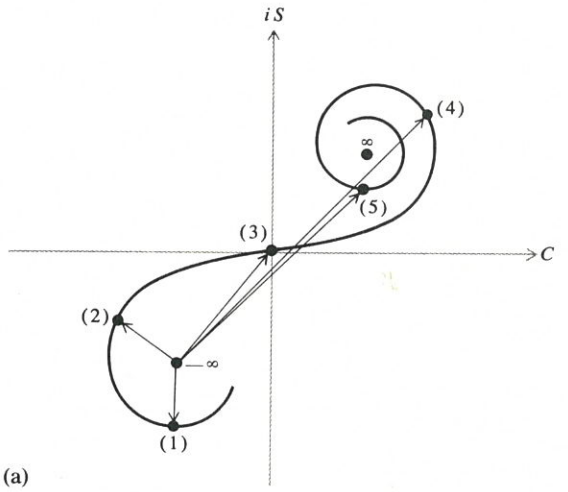
$$U_p = \frac{U_0}{(1+i)}(0.5+0.5i) = \frac{1}{2}U_0 .$$

$$I_p = |U_p|^2 = \frac{1}{4}I_0 ,$$

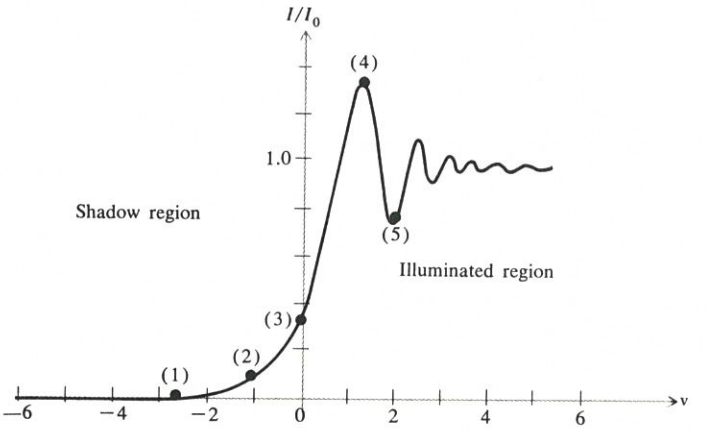
I_0 : unobstructed one,



morning glory



(a)



(b)

Figure 5.25. Fresnel diffraction by a straightedge. (a) Points on the Cornu spiral; (b) corresponding points on the intensity curve; $v = 0$ defines the geometrical shadow edge. A photograph of the diffraction pattern is shown below.