EC4214 Midterm exam Oct. 29, 2018

Name:

1. (10 points) The Jones matrix of a given polarizer is denoted by $\begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$.
   (i) Describe the Jones vector and polarization whose transmission is perfect to the Jones matrix.
   (ii) Describe the Jones vector and polarization whose transmission is zero to the given Jones matrix.
2. (20 points) At the Brewster's angle $\theta_B$, the reflection of a TM mode of polarized light is cancelled as shown in Fig. 1.
   (i) Prove that the reflected light has a right angle with respect to the transmitted light whose refraction angle is $\theta_t$. Hint: Fresnel Eqs.
   (ii) For a normal incidence, what is the reflectance $R_p$ for $n_1$=1 & $n_2$=1.5.
   (iii) (10 points) To reduce $R_p$ in (ii), anti-reflection coating is needed on top of $n_2$ material. What is the reflectance $R_p$ for the thickness t=$\pi$/2k and refractive index $n_3$ of the coating?

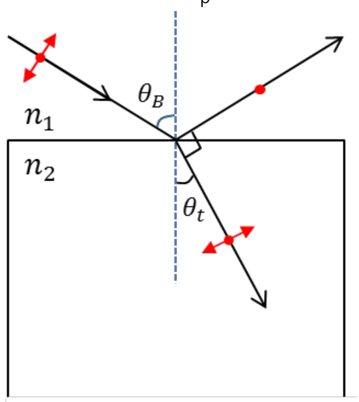



Fig. 1. Polarization at Brewster's angle.

3. (20 points) What is the transverse coherence width of sunlight? The apparent angular diameter of the sun is 0.5 degrees and the mean effective wavelength is 600 nm.
4. (15 points) In a Young's double slit experiment, whose wavelength of the monochromatic light is $\lambda$=500 nm, the slit distance is h=0.1 mm, and the screen-slit distance is x=1 m,
   (i) Describe the intensity $I_T$ of the light on the screen using symbols of h, x, $\lambda$, and y.
   (ii) Find out the position $y_0$ on the screen for the first minima of $I_T(y_0)$=0. Denote $y_0$ in terms of x, h, and $\lambda$.
   (iii) Discuss why the maximum $I_T$ does not violate the energy conservation law.
5. (15 points) A quarter-wave antireflecting film of magnesium fluoride (n=1.35) is coated on an optical glass surface of index 1.52.
   (i) Calculate the reflectance R.
   (ii) What is the condition for R=0?
6. (20 points) In a Fabry-Perot (FP) composed of two lossless parallel mirrors whose reflection coefficient is r, what is the reflectance R if FWHM of $I_T$ is 1% of the free spectral range of the FP? Hints: Fig. 4.1 & Fig. 4.3

Sol. #1

Jones matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \equiv M$

(i) For full transmission

$$(M)\begin{pmatrix} A \\ B \end{pmatrix} = \lambda \begin{pmatrix} A \\ B \end{pmatrix} \rightarrow \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \lambda \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\rightarrow \left.\begin{matrix} A + iB = \lambda A \\ -iA + B = \lambda B \end{matrix}\right) \rightarrow \begin{vmatrix} 1-\lambda & i \\ -i & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 1 = 0 \rightarrow 1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\rightarrow \lambda(\lambda - 2) = 0 \quad \therefore \lambda = 0 \ \& \ 2.$$

For $\lambda = 2$,

$$\left.\begin{matrix} A + iB = 2A \\ -iA + B = 2B \end{matrix}\right) \rightarrow \left.\begin{matrix} -A + iB = 0 \\ -iA - B = 0 \end{matrix}\right) \rightarrow \begin{matrix} -A + iB = 0 \\ A - iB = 0 \end{matrix}$$

$$\therefore \underline{A = iB}$$

$$\begin{pmatrix} i \\ 1 \end{pmatrix} = i\begin{pmatrix} 1 \\ -i \end{pmatrix}$$

The Jones vector is $\underline{\begin{pmatrix} 1 \\ -i \end{pmatrix}}$.

↳ right circularly polarized!

(ii) For zero transmission,

$$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \left[\begin{matrix} A + iB = 0 \\ -iA + B = 0 \rightarrow B = iA \end{matrix}\right. \rightarrow \underline{A = -iB}$$

$$\begin{pmatrix} -i \\ 1 \end{pmatrix} = -i\underline{\underline{\begin{pmatrix} 1 \\ i \end{pmatrix}}}$$

$\therefore$ Jones vector is $\underline{\begin{pmatrix} 1 \\ +i \end{pmatrix}}$.

↳ left circularly polarized!

Sol. # 2

(i) From Eq. (2.57), $r_p = 0$ (TM mode)

$$r_p = -\frac{\tan(\theta_B - \phi)}{\tan(\theta_B + \phi)} = 0 \quad ; \ \phi = \theta_t$$

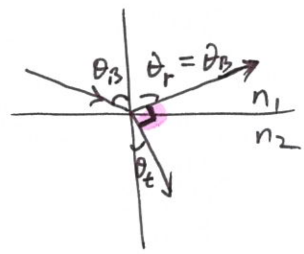
Due to Snell's law ($n_1 \sin\theta_B = n_2 \sin\phi$), $\theta \neq \phi$ due to $n_1 \neq n_2$.

$\therefore \tan(\theta_B + \phi) = \infty$ to make $r_p = 0$.

$\rightarrow \theta_B + \phi = \frac{\pi}{2}$ $\quad\therefore$ The angle btwn reflected and transmitted beams is a right angle.

(ii) From Eq. (2.59),

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \quad ; \ n = \frac{n_2}{n_1}$$

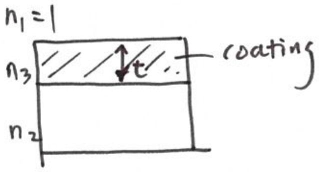



For normal incidence, $\theta = 0$.

$$\therefore r_p = \frac{-n^2 + n}{n^2 + n} = \frac{-n+1}{n+1}$$

$$R_p = |r_p|^2 = \left(\frac{-n+1}{n+1}\right)^2 = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = \underline{\underline{0.04}}$$

(iii)




from Eq. (4.32) $\quad r = \dfrac{n_3(1-n_2)\cos kt - i(n_2 - n_3^2)\sin kt}{n_3(1+n_2)\cos kt - i(n_2 + n_3^2)\sin kt}$

for $t = \frac{\pi}{2k}$,

$\cos kt = 0$ & $\sin kt = 1$. $\quad\therefore r = \dfrac{n_2 - n_3^2}{n_2 + n_3^2} \rightarrow \underline{\underline{R = \left(\dfrac{n_2 - n_3^2}{n_2 + n_3^2}\right)^2}}$.

Sol. #3

$$l_t = \frac{1.22\lambda}{\theta_s} \quad ), \quad \theta_s = 0.5^\circ \ \& \ \lambda = 600\,\text{nm}$$

$$= \frac{(1.22)(6\cdot 10^{-7})}{8.7\times 10^{-3}} = 8.4\times 10^{-5}\ \text{m}$$

$$= 84\ \mu\text{m}$$

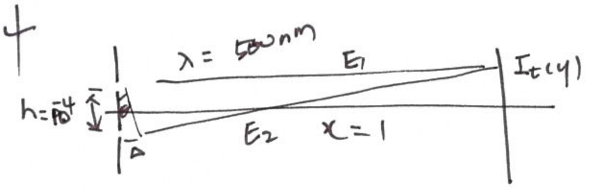
$$= 0.084\ \text{mm}$$

$$0.5 : 180 = x : \pi$$

$$x = \frac{0.5}{180}\cdot \pi = 8.7\times 10^{-3}$$

Sol. #4

(i)




$$E_T = E_1 + E_2$$

$$= E_0 (1 + e^{ik\Delta}) e^{i(kx - \omega t)}$$

$$I_T = |E_T|^2 = I_0 (1 + e^{ik\Delta})(1 + e^{-ik\Delta})$$

$$= I_0 (1 + 1 + e^{ik\Delta} + e^{-ik\Delta})$$

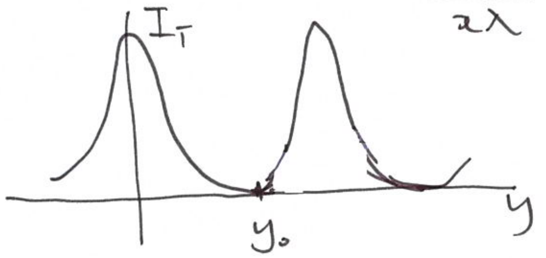
$$= 2I_0 (1 + \cos k\Delta).$$

$$\Delta = h\sin\theta \sim h\tan\theta = \frac{hy}{x}$$

$$\therefore \ I_T(y) = 2I_0 \left(1 + \cos\frac{khy}{x}\right) \qquad I_0 = |E_0|^2$$

$$= 2I_0 \left[1 + \cos\left(\frac{2\pi h y}{x\lambda}\right)\right]$$

(ii) $I_T(y) = 0. \ \rightarrow \ \dfrac{2\pi h y_0}{x\lambda} = \pi \ \rightarrow \ y_0 = \dfrac{x}{2h}\lambda$




(iii) The energy is the concept of average in time.

$$\langle I_T \rangle = \frac{1}{Y}\int_0^Y I_T\, dy = 2I_0, \quad \text{where} \quad \int \cos\left(\frac{2\pi y h}{x\lambda}\right) dy = 0.$$

This satisfies the energy conservation law!

Sol. # 5

(i) $R = |r|^2 = \dfrac{(n_T - n_1^2)^2}{(n_T + n_1^2)^2} = \left(\dfrac{1.52 - 1.35^2}{1.52 + 1.35^2}\right)^2 = \underline{\underline{8.2 \times 10^{-3}}}$

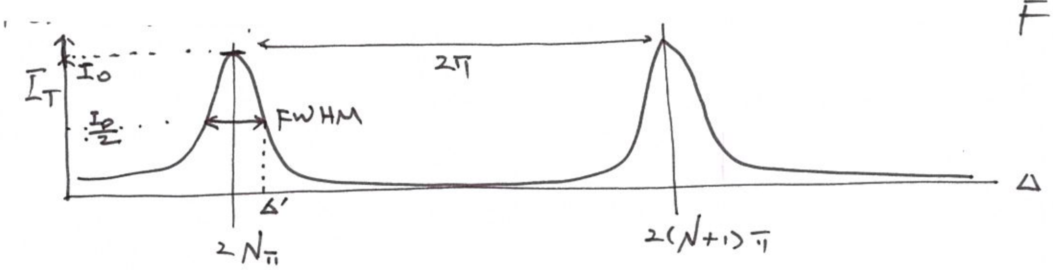
(ii) For $R = 0$, $\underline{n_T = \sqrt{n_1}}$.

Sol. #6

From Eq. (4.8)

$$I_T = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2\frac{\Delta}{2}} \Rightarrow I_0 \frac{1}{1 + F\sin^2\frac{\Delta}{2}},$$

$$F = \frac{4R}{(1-R)^2}$$




For FWHM, $I_T = \frac{1}{2} I_0 \longrightarrow F\sin^2\frac{\Delta'}{2} = 1$

Because FWHM is 1% of $2\pi$,

$\Delta' = \dfrac{\text{FWHM}}{2} = 0.01 \times 2\pi \longrightarrow \sin^2\dfrac{\Delta'}{2} = \sin^2\dfrac{(0.01 \times 2\pi)}{2}$

$= \sin^2(0.0314)$

$= \sin^2(1.8^\circ)$

$\fallingdotseq 0.001$

$\left( \begin{array}{l} 0.0314 : \pi = x : 180 \\ x = \dfrac{(0.0314)(180)}{\pi} = 1.8^\circ \end{array} \right.$

$\therefore F = 1000$

From $F = \dfrac{4R}{(1-R)^2} = 1000 \rightarrow (1 - 2R + R^2) \cdot 1000 = 4R$

$\rightarrow 1000R^2 - 2004R + 1000 = 0$

$\therefore R = \dfrac{2004 \pm \sqrt{2004^2 - 4000000}}{2000}$

$\fallingdotseq \dfrac{2004 \pm 126}{2000}$

$\therefore \underline{\underline{R = 0.939}}$