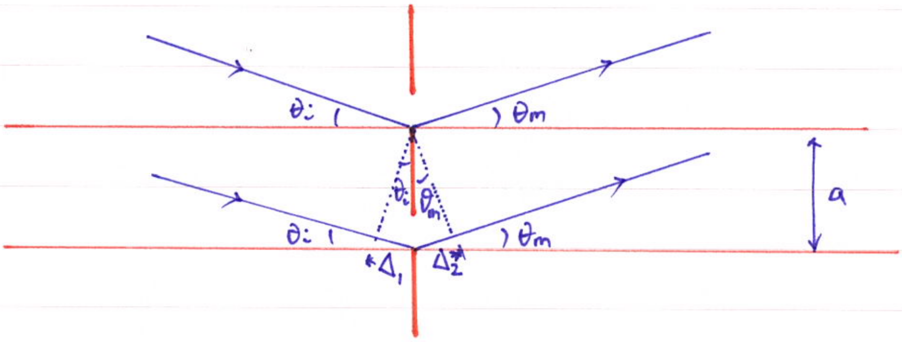
5/23/2016

CH.12 The Diffraction Grating

· N is large, → Generalized N-slit equation needed!

12-1. The grating equation



· Net path difference,

$$\Delta = \Delta_1 + \Delta_2 = a\sin\theta_i + a\sin\theta_m$$

Here, Δ_2 can be '+' or '−'.

→ If the incident & diffracted beams are on the same side : '+'.

· For constructive interference,

$$\Delta = m\lambda \quad , \quad m = 0, \pm 1, \pm 2, \cdots \quad : \text{Grating Eq.}$$

(i) zeroth order diffraction : $\theta_m = -\theta_i$ for all λ.

(ii) Higher order " : on both sides of the zeroth order.

12-2 Free Spectral Range of a Grating

· λ_{fsr} : non-overlapping λ range for m^{th} order

For the input light whose shortest wavelength is λ_1,

the nonoverlapping longest λ_2 for $(m+1)^{th}$ order of λ_1 is :

$$m\lambda_2 = (m+1)\lambda_1 \quad \rightarrow \quad \lambda_2 - \lambda_1 \equiv \lambda_{fsr} = \frac{\lambda_1}{m}$$

ex) Shorest wavength : 400 nm for m^{th} order grating. Determine λ_{fsr} in the first 3-orders.

Sol) $\lambda_{fsr} = \dfrac{\lambda_1}{m}$

(i) For $m=1$; $\lambda_{fsr} = \lambda_1 = 400$ nm $\rightarrow \lambda_2 = 800$ nm
(ii) For $m=2$; $\lambda_{fsr} = \lambda_1/2 = 200$ nm $\rightarrow \lambda_2 = 600$ nm
(iii) For $m=3$; $\lambda_{fsr} = \lambda_1/3 = 133$ nm $\rightarrow \lambda_2 = 533$ nm

12-3 Dispersion of a Grating

- From ex), at higher order, it gives better resolution.

- Angular Dispersion, $\mathcal{D}$

$$\mathcal{D} \equiv \frac{d\theta_m}{d\lambda}.$$

$\rightarrow$ From Grating Eq, $a\sin\theta_m = m\lambda$

$\rightarrow a\cos\theta_m \, d\theta_m = m\, d\lambda$

$$\therefore \frac{d\theta_m}{d\lambda} = \frac{m}{a\cos\theta_m}.$$

- Linear Dispersion, $\frac{dy}{d\lambda}$

$$\rightarrow \frac{dy}{d\lambda} = f\frac{d\theta_m}{d\lambda} = f\mathcal{D}. \qquad (\equiv 1/\text{plate factor})$$

PAGE

DATE

ex) $\lambda = 500\,nm$, incident normally on a grating.

· grating : 5000 groves/cm

Determine the angular and linear dispersion in first order, when a $f = 50cm$ lens is used.

Sol). $\mathcal{D} = \frac{m}{a\cos\theta_m} = \frac{1}{(2\cdot10^{-4}(cm))(0.97)} = 5165\ (rad/cm)$

$a = \frac{0.01(m)}{5000} = 2\times10^{-4}\ (cm)$ &

$a\sin\theta_m = m\lambda \rightarrow \sin\theta_1 = \frac{\lambda}{a} = \frac{5\cdot10^{-7}}{2\cdot10^{-6}} = \underline{0.25}$

$\therefore \theta_1 = \sin^{-1}(0.25) = 14.5°$

$\rightarrow \cos\theta_1 = 0.97$

linear dispersion : $f\mathcal{D}$

$f\mathcal{D} = 0.5\,(m)\,(5165\,(rad/cm)) = 2582\ m/cm$

$= 258.2\ mm/nm = 0.258\ mm/nm$

✱ From $\mathcal{D} = \frac{m}{a\cos\theta_m}$ & $a\sin\theta_m = m\lambda$

$\mathcal{D} = \frac{a\sin\theta_m}{a\cos\theta_m}\frac{1}{\lambda} = \frac{\tan\theta_m}{\lambda}$

$\therefore \mathcal{D}$ is independent of a.

$\uparrow$

grating const.

$\therefore \theta_m \uparrow,\ \mathcal{D}\uparrow$

12-4. Resolution of a Grating

From CH. 8,

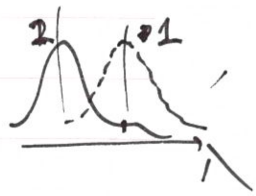
Resolving power of Fabry-Perot is

$$R \equiv \frac{\lambda}{(\Delta\lambda)_{min}}.$$

For the diffraction a m^{th} order by $\lambda + d\lambda$ is

$$a \sin\theta_m = m(\lambda + d\lambda) \qquad ①$$

Rayleigh's criterion, $Max^{(2)} = Min^{(1)}$



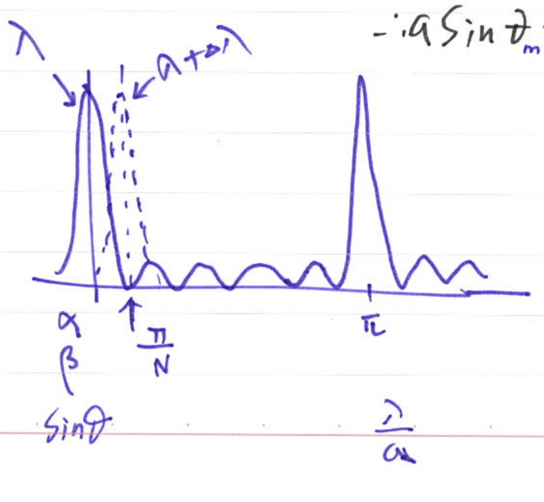
· From N-slit diffraction grating,

the condition for the zeros resulting from interference effect $\left(\frac{\sin N\alpha}{\sin\alpha}\right)$ is

$$N\alpha = p\pi \rightarrow \alpha = \frac{\pi}{N} \quad (\text{1st min.}) \quad \text{for } \lambda + d\lambda$$

$$\alpha = \frac{1}{2}k' a \sin\theta_m \rightarrow (\sin\theta_m)\, a \left(\frac{\cancel{2\pi}}{\cancel{2}\lambda'}\right) = \frac{\cancel{\pi}}{N}$$

$$\therefore a \sin\theta_m = \underbrace{\frac{\lambda'}{N}}_{\Rightarrow m\, d\lambda} \quad (\text{for } \overbrace{\lambda + d\lambda}^{=\lambda'}) \qquad ②$$



$$\left[a \sin\theta_m = \left(m + \frac{1}{N}\right)\lambda \right]$$

$$\therefore \frac{\lambda}{d\lambda} = mN \equiv R$$

resolving power!

PAGE
DATE

ex) · 5000 grooves/cm · Total width of a grating : 8 cm
· $\lambda = 500$ nm

sol) $N = ?$, $d\lambda = ?$, $R = ?$

(i) $N = 5000 \times 8 = 40{,}000$

For $m = 1$,

(ii) $$\frac{\lambda}{N} = a\sin\theta_1 = 1 \cdot d\lambda$$

$$\therefore d\lambda = \frac{500 \cdot 10^{-9}}{40{,}000} = 1.25 \times 10^{-11} = 12.5 \text{ pm}$$

(iii) $$R = \frac{\lambda}{d\lambda} = \frac{500{,}000}{12.5} = 40{,}000 \quad (\text{for } m=1)$$

cf) Fabry-Perot Resolving power (CH. 8)

$$R = \frac{\lambda}{\Delta\lambda_{min}} = m\mathcal{F} \quad , (\mathcal{F} = \text{finesse})$$

$\sim 10^{6\sim9}$

ex) $R = 10^6$ for $m = 5$.
10,000 grooves/cm
20-cm width grating

Q1. What is maximum λ for normal & blazed θ_i?

A1. $a(\sin\theta_i + \sin\theta_m) = m\lambda \Rightarrow 5\lambda$

$$a = \frac{1\,cm}{10{,}000} = 10^{-6}\,(m) \quad ; \quad N = 10{,}000 \times 20 = 2\times10^{5}$$

(i) For normal incidence, $\theta_i = 0$,

$$a\sin\theta_{m=5} = 5\lambda$$

For maximum λ, $\sin\theta_{m=5} = 1$

$$\therefore \lambda = \frac{a}{5} = 200\ nm$$

(ii) For a blazing angle incidence, $\theta_i = 90^\circ$,

$$a(\sin\theta_{in} + \sin\theta_m) = m\lambda$$

$$\rightarrow a(1+1) = 5\lambda$$

$$\therefore \lambda = \frac{2}{5}a = 400\,(nm).$$

<Comparison>

	Fabry-Perot	Diffraction Grating
R (resolving power)	$m\mathcal{F}$	mN
$\Delta\lambda_{min} = \frac{\lambda}{R}$ (min. resolvable)	$\frac{\lambda}{m\mathcal{F}}$	$\frac{\lambda}{mN}$
λ_{fsr}	$\frac{\lambda_1}{m}$	$\frac{\lambda_1}{m}$

· Resolving power R is independent of groove spacing (a) for a given diffraction angle: $R = mN$

* For a width, W,

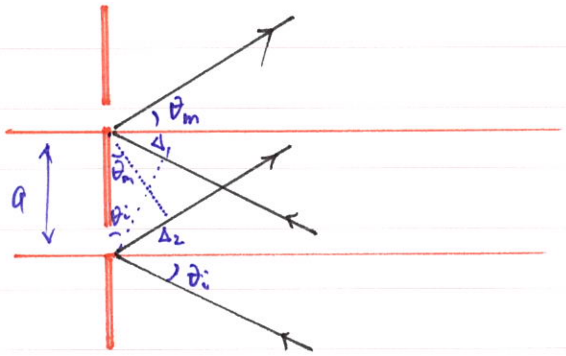
→ $N = W/a$; $a\sin\theta_m = m\lambda$

$$R = mN = \left(\frac{a\sin\theta_m}{\lambda}\right)\left(\frac{W}{a}\right) = \frac{W\sin\theta_m}{\lambda}.$$

∴ Resolution of a grating at diffraction angle θ_m depends on the width of grating rather than the number of its grooves!

12-5 Types of Gratings

$\left\langle \begin{array}{l} \text{Transmission} \left\langle \begin{array}{l} \text{Transmission amplitude grating} \\ \text{" phase grating} \end{array} \right. \\ \text{Reflection} \end{array} \right.$



$$\Delta = \Delta_1 - \Delta_2 = a\sin\theta_i - a\sin\theta_m$$

· For principal maxima: $m\lambda = a(\sin\theta_i + \sin\theta_m)$

· Sign convention: '+' for the same side of the grating normal.

· zeroth order: at $\theta_m = -\theta_i$ → mirror effect.

· Coating materials: Metal (MgF, LF) on Al

Au, Pt for $\lambda < 100$ nm